## Genetic Algorithm Example:

Evolving a Control Program for a Virtual "Robot"

## Herbert:

The Soda Can Collecting Robot (Connell, Brooks, Ning, 1988)

http://cyberneticzoo.com/?p=5516

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## Robby:

The Virtual Soda Can Collecting Robot (Mitchell, 2009)


Robby's World

| Soda can | $\mathfrak{q}_{0}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 5 |  | $\bigcirc$ | 5 |  |  |  |  |  |
|  |  |  |  | 9 |  |  | $\bigcirc$ |  |  |  |
|  |  |  | 5 |  |  |  |  |  |  |  |
| 3 |  | 9 |  | 0 |  |  |  |  |  |  |
| 4 |  |  |  |  |  | 5 |  | 0 |  |  |
| 5 |  |  |  |  |  |  |  |  |  |  |
| 6 |  |  | 0 | 9 |  | $0$ |  |  | 5 |  |
| 7 |  |  |  | 8 |  |  |  | $\bigcirc$ |  |  |
| 8 |  |  |  |  |  | 5 |  |  |  |  |
| 9 |  | 0 |  |  |  |  |  |  |  |  |

## What Robby Can See and Do

## Input:

Contents of North, South, East, West, Current

## Possible actions:

Move N
Move S
Move E
Move W
Move random
Stay put
Try to pick up can


Rewards/Penalties (points):
Picks up can: 10
Tries to pick up can on empty site: $\mathbf{- 1}$
Crashes into wall: -5
Robby's Score: Sum of rewards/penalties

Goal: Use a genetic algorithm to evolve a control program (i.e., strategy) for Robby.

## One Example Strategy

|  | Situation |  |  |  |  | Action |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | North | South | East | West | Current Site |  |  |  |  |  |  |  |  |  |  |  |
| 1 | Empty | Empty | Empty | Empty | Empty | MoveNorth |  |  |  |  |  |  |  |  |  |  |
| 2 | Empty | Empty | Empty | Empty | Can | MoveEast |  |  |  |  |  |  |  |  |  |  |
| 3 | Empty | Empty | Empty | Empty | Wall | MoveRandom | 9 |  |  |  |  |  |  |  |  |  |
| 4 | Empty | Empty | Empty | Can | Empty | PickUpCan |  |  | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|  |  |  |  |  |  |  |  | - |  | - | - |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  | - |  |  | $\bigcirc$ |  |  |  |
| . | Wall | Empty | Can | Wall | Empty | MoveWest |  |  | I |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  | [ |  | $\square$ |  |  |  |  |  |  |
| 243 | Wall | Wall | Wall | Wall | Wall | StayPut |  |  |  |  |  | $\square$ |  | $\bullet$ |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | 1 | - |  | - |  |  | $\bigcirc$ |  |
|  |  |  |  |  |  |  | - |  |  | $\square$ |  |  |  | $\triangle$ |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  | $\square$ |  |  |  |  |
|  |  |  |  |  |  |  |  | E |  |  |  |  |  |  |  |  |

## Encoding a Strategy



## Action

| 1 | MoveNorth |
| :--- | :---: |
| 2 | MoveEast |
| 3 | MoveRandom |
| 4 | PickUpCan |
| . | $\vdots$ |
| . | MoveWest |
| . | $\vdots$ |
|  | StayPut |

## Action

| 1 | MoveNorth |
| :--- | :---: |
| 2 | MoveEast |
| 3 | MoveRandom |
| 4 | PickUpCan |
| . | $\vdots$ |
| - | MoveWest |
| . | $\vdots$ |
|  | StayPut |

Code:
MoveNorth $=0$
MoveSouth $=1$
MoveEast $=2$
MoveWest $=3$
StayPut = 4
PickUpCan = 5
MoveRandom $=6$

## Action



Question: How many possible strategies are there in our representation?

## 0265 ... 3 ... 4

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$\longleftarrow 243$ values $\longrightarrow$

## 0265 ... 3 ... 4

7 possible actions for each position:

$$
7 \times 7 \times 7 \times \ldots
$$

$\times 7$

Question: How many possible strategies are there in our representation?
$\longleftarrow 243$ values $\longrightarrow$

## 0265 ... 3 ... 4

7 possible actions for each position:

$$
7 \times 7 \times 7 \times \ldots
$$

$\times 7$

Goal: Have GA search intelligently in this vast space for a good strategy

## Genetic algorithm for evolving strategies

1. Generate 200 random strategies (i.e., programs for controlling Robby)
2. For each strategy, calculate fitness (average reward minus penalties earned on random environments)
3. The strategies pair up and create offspring via crossover with random mutations - the fitter the parents, the more offspring they create.
4. Keep going back to step 2 until a good-enough strategy is found (or for a set number of generations)

## Robby's fitness function

```
Calculate_Fitness (Robby) {
    Total_Reward = 0 ;
    Average_Reward = 0 '
    For i = 1 to NUM_ENVIRONMENTS {
        generate_random_environment( ); /* . 5 probability 
        For j = 1 to NUM_MOVES_PER_ENVIRONMENT {
        Total_Reward = Total_Reward + perform_action(Robby);
        }
    }
    Fitness = Total_Reward / NUM_ENVIRONMENTS;
    return(Fitness);
}
```


## Genetic algorithm for evolving strategies for Robby

1. Generate 200 random strategies (i.e., programs for controlling Robby)

## Random Initial Population

```
Individual 1:
23300323421630343530546006102562515114162260435654334066511514 15650220640642051006643216161521652022364433363346013326503000 40622050243165006111305146664232401245633345524126143441361020 150630642551654043264463156164510543665346310551646005164
Individual 2:
16411343121025360340361241431201104235462525304202044516433665 61035322153105131440622120614631432154610256523644422025340345 30502005620634026331002453416430151631210012214400664012665246 351650154123113132453304433212634555005314213064423311000
Individual 3:
20423344402411226132136452632464212206122122252660626144436125 32512664061335340153411110206164226653145522540234051155031302 22020065445125062206631426135532010000400031640130154160162006 134440626160505641421553133236021503355131253632642630551
```

Individual 200:
34632525136001012225612106043301135205155320130656005322235043 32425064124255265534635345523053326612010632124554423440613654 30246240160663016464641103026540006334126150352262106063624260 550616616344255124354464110023463330440102533212142402251

## Genetic algorithm for evolving strategies for Robby

1. Generate 200 random strategies (i.e., programs for controlling Robby)
2. For each strategy in the population, calculate fitness (average reward minus penalties earned on random environments)


Fitness =
Average final score from N moves on each of M random environments

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3. Strategies are selected according to fitness to become parents. (See code for choice of selection methods.)

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1. Generate 200 random strategies (i.e., programs for controlling Robby)
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3. Strategies are selected according to fitness to become parents. (See code for choice of selection methods.)
4. The parents pair up and create offspring via crossover with random mutations.

## Parent 1:

16411343121025360340361241431201104235462525304202044516433665 61035322153105131440622120614631432154610256523644422025340345 30502005620634026331002453416430151631210012214400664012665246 351650154123113132453304433212634555005314213064423311000

## Parent 2:

20423344402411226132136452632464212206122122252660626144436125 32512664061335340153411110206164226653145522540234051155031302 22020065445125062206631426135532010000400031640130154160162006 134440626160505641421553133236021503355131253632642630551

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## Parent 2:

20423344402411226132136452632464212206122122252660626144436125 32512664061335340153411110206164226653145522540234051155031302 $220200654451250622066314 i 6135532010000400031640130154160162006$ 134440626160505641421553133236021503355131253632642630551

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16411343121025360340361241431201104235462525304202044516433665 61035322153105131440622120614631432154610256523644422025340345 30502005620634026331002453416430151631210012214400664012665246 351650154123113132453304433212634555005314213064423311000

## Parent 2:

20423344402411226132136452632464212206122122252660626144436125 32512664061335340153411110206164226653145522540234051155031302 $220200654451250622066314 ; 6135532010000400031640130154160162006$ 134440626160505641421553133236021503355131253632642630551

## Child:

16411343121025360340361241431201104235462525304202044516433665 61035322153105131440622120614631432154610256523644422025340345 30502005620634026331002456135532010000400031640130154160162006 134440626160505641421553133236021503355131253632642630551

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16411343121025360340361241431201104235462525304202044516433665 61035322153105131440622120614631432154610256523644422025340345 30502005620634026331002453416430151631210012214400664012665246 351650154123113132453304433212634555005314213064423311000

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20423344402411226132136452632464212206122122252660626144436125 32512664061335340153411110206164226653145522540234051155031302 $220200654451250622066314 ; 6135532010000400031640130154160162006$ 134440626160505641421553133236021503355131253632642630551

## Child:

## Mutate to "0"

16411343121025360340361241431201104235462525304202044516433665 61035322153105131440622120614631432154610256523644422025340345 30502005620634026331002456135532010000400031640130154160162006 134440626160505641421553133236021503355131253632642630551

$$
\text { Mutate to " } 4 "
$$

## Genetic algorithm for evolving strategies for Robby

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5. The offspring are placed in the new population and the old population dies.
6. Keep going back to step 2 until a good-enough strategy is found!

My hand-designed strategy:
"If there is a can in the current site, pick it up."
"Otherwise, if there is a can in one of the adjacent sites, move to that site."
"Otherwise, choose a random direction to move in, avoiding walls."

My hand-designed strategy:
"If there is a can in the current site, pick it up."
"Otherwise, if there is a can in one of the adjacent sites, move to that site."
"Otherwise, choose a random direction to move in,
Average fitness of this strategy: 346
(out of max possible $\approx 500$ )

My hand-designed strategy:
"If there is a can in the current site, pick it up."
"Otherwise, if there is a can in one of the adjacent sites, move to that site."
"Otherwise, choose a random direction to move in,

Average fitness of this strategy: 346
(out of max possible $\approx 500$ )

Average fitness of GA evolved strategy: 486
(out of max possible $\approx 500$ )

## One Run of the Genetic Algorithm



## Generation 1

Best fitness $=-81$

Time: $0 \quad$ Score: 0

| S |  | - |  |  |  |  | 1 | 1 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | - | - | 1 | 1 |  |  |  |  |
| $\square$ |  | 1 | $\bullet$ |  | 1 | 1 | 1 |  |  |
|  |  |  | 1 | $\bullet$ | - | 1 |  |  |  |
|  |  |  |  | 1 | 1 | 1 |  |  |  |
|  |  |  | $\square$ |  | 1 |  | 1 | $\square$ |  |
| 1 | 1 | $\square$ |  |  | 1 | 1 | 1 | 1 | - |
| 1 | 1 |  |  |  | 1 | 1 | 1 |  | - |
| $\square$ |  |  | 1 | , | , | - | - |  |  |
| 1 | - |  |  |  |  |  |  |  |  |

## Time: 1 Score: 0



Time: 2 Score: -5


Time: 2 Score: -5


Time: 3 Score: -10


Time: 3 Score: -10


Time: 4 Score: -15


## Time: 4 Score: -15

| 4 |  | - |  |  |  |  | 1 | $\square$ | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 |  | $\square$ | - | - | U |  |  |  |  |
| - |  | 1 | - |  | 1 | 1 | 1 |  |  |
|  |  |  | - | 1 | - | - |  |  |  |
| 4 |  |  |  | 1 | 1 | 1 |  |  |  |
| 5 |  |  | - |  | 1 |  | 1 | 1 |  |
| - | 1 | $\square$ |  |  | $\bullet$ | $\bullet$ | 1 | 1 | 1 |
| 1 | 1 |  |  |  | 1 | 1 | 1 |  | 1 |
| 1 |  |  | - | 1 | 1 | 1 | - |  |  |
| $\square$ | 1 |  |  |  |  |  | - | 1 | 1 |

## Generation 14

## Best fitness $=1$

Time: $0 \quad$ Score: 0

| S |  | - |  |  |  |  | 1 | 1 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | - | - | 1 | 1 |  |  |  |  |
| $\square$ |  | 1 | $\bullet$ |  | 1 | 1 | 1 |  |  |
|  |  |  | 1 | $\bullet$ | - | 1 |  |  |  |
|  |  |  |  | 1 | 1 | 1 |  |  |  |
|  |  |  | $\square$ |  | 1 |  | 1 | $\square$ |  |
| 1 | 1 | $\square$ |  |  | 1 | 1 | 1 | 1 | - |
| 1 | 1 |  |  |  | 1 | 1 | 1 |  | - |
| $\square$ |  |  | 1 | , | , | - | - |  |  |
| 1 | - |  |  |  |  |  |  |  |  |

Time: 1 Score: 0

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| , |  | - |  |  |  |  | - | 1 | - |
| 1 |  | $\bigcirc$ | 1 | 1 | 1 |  |  |  |  |
| 1 |  | - | - |  | 1 | 1 | $\square$ |  |  |
|  |  |  | 1 | 1 | 1 | - |  |  |  |
|  |  |  |  | 1 | $\square$ | $\square$ |  |  |  |
|  |  |  | 1 |  | 1 |  | - | 1 |  |
| 1 | 1 | 1 |  |  | $\square$ | $\square$ | 1 | 1 | 1 |
| 1 | 1 |  |  |  | 1 | 1 | 1 |  | - |
| $\square$ |  |  | - | - | 1 | - | - |  |  |
| 1 | 1 |  |  |  |  |  | 1 | 1 | 1 |

Time: 2 Score: 0

|  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| S |  | $\square^{1}$ |  |  |  |  | 1 | 1 |  |
| 1 |  | 1 | - | - | 1 |  |  |  |  |
| 1 |  | 1 | 1 |  | 1 | 1 | 1 |  |  |
|  |  |  | 1 | - | 1 | 1 |  |  |  |
|  |  |  |  | 1 | 1 | 1 |  |  |  |
|  |  |  | $\square$ |  | - |  | 1 | 1 |  |
| - | 1 | 1 |  |  | 1 | - | 1 | - | $\square$ |
| - | 1 |  |  |  | 1 | 1 | 1 |  | - |
| $\square$ |  |  | $\square$ | J | 1 | 0 | 1 |  |  |
| $\square$ | 1 |  |  |  |  |  |  |  |  |

Time: 3 Score: 0

|  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| S |  | - |  |  |  |  | 1 | 1 |  |
| $\square$ |  | - | 1 | - | 1 |  |  |  |  |
| - |  | 1 | $\square$ |  | $\square$ | 1 | 1 |  |  |
|  |  |  | $\bullet$ | 1 | $\bullet$ | - |  |  |  |
|  |  |  |  | 1 | - | 1 |  |  |  |
|  |  |  | 1 |  | 1 |  | 1 | 1 |  |
| - | - | 1 |  |  | $\square$ | - | $\square$ | - |  |
| - | 1 |  |  |  | $\pm$ | 1 | 1 |  |  |
| 8 |  |  | 1 | 1 | 1 | 8 | 1 |  |  |
| $\square$ | 1 |  |  |  |  |  | 1 | 1 |  |

# Generation 200 

Fitness $=240$

Time: $0 \quad$ Score: 0

| S |  | - |  |  |  |  | 1 | 1 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | - | - | 1 | 1 |  |  |  |  |
| $\square$ |  | 1 | $\bullet$ |  | 1 | 1 | 1 |  |  |
|  |  |  | 1 | $\bullet$ | - | 1 |  |  |  |
|  |  |  |  | 1 | 1 | 1 |  |  |  |
|  |  |  | $\square$ |  | 1 |  | 1 | $\square$ |  |
| 1 | 1 | $\square$ |  |  | 1 | 1 | 1 | 1 | - |
| 1 | 1 |  |  |  | 1 | 1 | 1 |  | - |
| $\square$ |  |  | 1 | , | , | - | - |  |  |
| 1 | - |  |  |  |  |  |  |  |  |

## Time: 1 Score: 0



Time: 2 Score: 0


Time: 3 Score: 10

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  | $\checkmark$ |  |  |  |  | $\bigcirc$ | 0 | $\bigcirc$ |
|  | 8 |  | 1 | - | $\bigcirc$ | 0 |  |  |  |  |
| 2 | $\triangle$ |  | 1 | - |  | $\bigcirc$ | $\bigcirc$ | $\square$ |  |  |
| 3 |  |  |  | - | $\square$ | $\square$ | 1 |  |  |  |
| 4 |  |  |  |  | $\checkmark$ | 0 | $\square$ |  |  |  |
| 5 |  |  |  | - |  | $\square$ |  | 0 | 0 |  |
| 6 | $\square$ | 1 | - |  |  | 0 | - | $\square$ | 0 | 0 |
| 7 | $\square$ | - |  |  |  | 1 | - | - |  | 1 |
| 8 | $\square$ |  |  | - | - | $\bigcirc$ | $\bigcirc$ | $\triangle$ |  |  |
| 9 | $\square$ | $\square$ |  |  |  |  |  | 8 | 0 | $\square$ |

Time: 4 Score: 10


Time: 5 Score: 20


Time: 6 Score: 20

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  | $\checkmark$ |  |  |  |  | 1 | 1 | $\bigcirc$ |
| 1 | 9 |  | 1 | 1 | 1 | $\square$ |  |  |  |  |
| 2 | 而 |  | 1 | 1 |  | - | 1 | 1 |  |  |
| 3 |  |  |  | 1 | - | $\bigcirc$ | 1 |  |  |  |
| 4 |  |  |  |  | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |  |  |  |
| 5 |  |  |  | 1 |  | 0 |  | 8 | - |  |
| 6 | 0 | 1 | 1 |  |  | - | 0 | 0 | - | 0 |
| 7 | $\square$ | - |  |  |  | 1 | 1 | 1 |  | 1 |
| 8 | $\bigcirc$ |  |  | $\bigcirc$ | $\bigcirc$ | 1 | 1 | 1 |  |  |
| 9 | $\square$ | $\square$ |  |  |  |  |  | $\square$ | $\bigcirc$ | $\square$ |

Time: 7 Score: 20


Time: 8 Score: 20

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  | $\checkmark$ |  |  |  |  | 1 | $\bigcirc$ | $\bigcirc$ |
| 1 |  |  | - | - | - | 0 |  |  |  |  |
| 2 |  |  | - | - |  | $\bigcirc$ | - | 1 |  |  |
| 3 | 9 |  |  | $\bigcirc$ | $\square$ | $\square$ | $\square$ |  |  |  |
| 4 | $\cdots$ |  |  |  | $\square$ | $\bigcirc$ | $\square$ |  |  |  |
| 5 |  |  |  | 1 |  | $\square$ |  | $\square$ | $\bigcirc$ |  |
| 6 | 1 | 1 | - |  |  | 0 | $\square$ | 0 | $\bigcirc$ | $\square$ |
| 7 | 1 | $\square$ |  |  |  | 1 | - | 1 |  | $\square$ |
| 8 | $\square$ |  |  | $\square$ | $\square$ | $\bigcirc$ | $\square$ | $\bigcirc$ |  |  |
| 9 | $\bigcirc$ | - |  |  |  |  |  | 1 | 0 | 0 |

Time: 9 Score: 20

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  | $\checkmark$ |  |  |  |  | 1 | $\bigcirc$ | $\bigcirc$ |
| 1 |  |  | - | - | - | 0 |  |  |  |  |
| 2 |  |  | $\bigcirc$ | - |  | $\bigcirc$ | $\square$ | 1 |  |  |
| 3 |  |  |  | 1 | 1 | $\bigcirc$ | - |  |  |  |
| 4 | 9 |  |  |  | $\square$ | $\checkmark$ | $\square$ |  |  |  |
| 5 |  |  |  | $\bigcirc$ |  | $\bigcirc$ |  | - | $\bigcirc$ |  |
| 6 | $\bigcirc$ | 1 | - |  |  | 0 | $\square$ | 0 | $\bigcirc$ | $\square$ |
| 7 | $\bigcirc$ | $\square$ |  |  |  | $\bigcirc$ | $\square$ | $\bigcirc$ |  | $\square$ |
| 8 | $\square$ |  |  | $\square$ | $\square$ | $\bigcirc$ | $\square$ | $\bigcirc$ |  |  |
| 9 | $\square$ | $\square$ |  |  |  |  |  | $\square$ | $\square$ | $\square$ |

Time: 10 Score: 20

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  | $\checkmark$ |  |  |  |  | 1 | $\bigcirc$ | $\bigcirc$ |
| 1 |  |  | 0 | - | $\square$ | 0 |  |  |  |  |
| 2 |  |  | $\bigcirc$ | $\bigcirc$ |  | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |  |  |
| 3 |  |  |  | 1 | 1 | $\bigcirc$ | 1 |  |  |  |
| 4 |  |  |  |  | 1 | 0 | - |  |  |  |
| $5$ | $9$ |  |  | 1 |  | $\square$ |  | $\square$ | $\bigcirc$ |  |
| 6 | - | 1 | $\bigcirc$ |  |  | $\bigcirc$ | $\square$ | $\square$ | $\bigcirc$ | $\square$ |
| 7 | 1 | $\square$ |  |  |  | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |  | $\square$ |
| 8 | - |  |  | - | $\square$ | 0 | - | $\square$ |  |  |
| 9 | $\square$ | $\bigcirc$ |  |  |  |  |  | $\square$ | $\square$ | $\square$ |

Time: 11 Score: 20


Time: 12 Score: 20

|  |  | 1 |  |  |  |  | 1 | 1 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | - | - | 1 |  |  |  |  |
|  |  | 1 | 1 |  | $\square$ | 1 | 1 |  |  |
|  |  |  | $\bullet$ | $\square$ | - | $\bullet$ |  |  |  |
|  |  |  |  | 1 | - | - |  |  |  |
|  |  |  | 1 |  | 1 |  | - | 1 |  |
| 1 | 1 | 1 |  |  | $\square$ | 1 | 1 | 1 |  |
| 9 | - |  |  |  | $\bullet$ | 1 | 1 |  |  |
| T |  |  | - | - | 1 | 1 | 1 |  |  |
| 0 | 1 |  |  |  |  |  | 1 | 1 |  |

Time: 13 Score: 20

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  | $\checkmark$ |  |  |  |  | 1 | 1 | $\square$ |
| 1 |  |  | 1 | $\square$ | 0 | $\square$ |  |  |  |  |
| 2 |  |  | $\square$ | $\square$ |  | $\square$ | 1 | 1 |  |  |
| 3 |  |  |  | $\square$ | $\square$ | 0 | 1 |  |  |  |
| 4 |  |  |  |  | 1 | $\square$ | 1 |  |  |  |
| 5 |  |  |  | $\square$ |  | $\bigcirc$ |  | - | $\bigcirc$ |  |
| 6 | $\square$ | 1 | $\bigcirc$ |  |  | $\bigcirc$ | $\bigcirc$ | - | $\bigcirc$ | 1 |
| 7 | $\bigcirc$ | $\square$ |  |  |  | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |  | 0 |
| 8 |  |  |  | $\square$ | - | - | $\bigcirc$ | $\square$ |  |  |
| 9 | 20 | - |  |  |  |  |  | 1 | 0 | $\square$ |

Time: 14 Score: 30


Time: 15 Score: 30

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  | $\checkmark$ |  |  |  |  | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 1 |  |  | 0 | - | $\square$ | 0 |  |  |  |  |
| 2 |  |  | - | $\square$ |  | $\bigcirc$ | 1 | $\square$ |  |  |
| 3 |  |  |  | $\bigcirc$ | $\square$ | $\square$ | - |  |  |  |
| 4 |  |  |  |  | 1 | 0 | - |  |  |  |
| 5 |  |  |  | $\bigcirc$ |  | $\bigcirc$ |  | 0 | $\bigcirc$ |  |
| 6 | $\square$ | 1 | 0 |  |  | $\bigcirc$ | $\bigcirc$ | 0 | $\bigcirc$ | $\square$ |
| $7$ | 9 | $\square$ |  |  |  | 1 | 1 | - |  | $\square$ |
| 8 | 1 |  |  | $\bigcirc$ | 1 | $\bigcirc$ | - | $\bigcirc$ |  |  |
| 9 |  | $\square$ |  |  |  |  |  | $\square$ | $\square$ | $\square$ |

Time: 16 Score: 40

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  | $\checkmark$ |  |  |  |  | 1 | $\bigcirc$ | $\bigcirc$ |
| 1 |  |  | - | - | - | 0 |  |  |  |  |
| 2 |  |  | $\bigcirc$ | $\bigcirc$ |  | $\bigcirc$ | $\square$ | $\bigcirc$ |  |  |
| 3 |  |  |  | - | $\square$ | $\square$ | $\square$ |  |  |  |
| 4 |  |  |  |  | 1 | 0 | - |  |  |  |
| 5 |  |  |  | $\bigcirc$ |  | $\square$ |  | $\bigcirc$ | $\bigcirc$ |  |
| 6 | 1 | 1 | $\bigcirc$ |  |  | $\bigcirc$ | $\square$ | $\bigcirc$ | $\bigcirc$ | $\square$ |
| 7 | 9 | - |  |  |  | $\bigcirc$ | $\square$ | $\bigcirc$ |  | $\square$ |
| 8 | C |  |  | $\square$ | $\square$ | $\bigcirc$ | $\square$ | $\bigcirc$ |  |  |
| 9 |  | - |  |  |  |  |  | $\square$ | $\square$ | $\square$ |

Time: 17 Score: 40


Time: 18 Score: 50


Time: 19 Score: 50


Time: 20 Score: 60

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  | $\checkmark$ |  |  |  |  | 1 | $\bigcirc$ | $\bigcirc$ |
| 1 |  | - | $\bigcirc$ | - | 0 |  |  |  |  |
| 2 |  | $\bigcirc$ | 1 |  | $\bigcirc$ | $\square$ | 1 |  |  |
| 3 |  |  | - | $\square$ | $\square$ | $\square$ |  |  |  |
| 4 |  |  |  | 1 | 0 | - |  |  |  |
| 5 |  |  | 1 |  | $\square$ |  | $\square$ | $\bigcirc$ |  |
| 6 | $\square$ | $\bigcirc$ |  |  | $\bigcirc$ | $\square$ | $\square$ | $\bigcirc$ | $\square$ |
| 7 | $\bigcirc$ |  |  |  | $\bigcirc$ | $\square$ | $\bigcirc$ |  | $\square$ |
| 8 |  |  | - | 1 | $\bigcirc$ | $\square$ | 1 |  |  |
| 9 | $\square$ |  |  |  |  |  | $\square$ | $\square$ | $\square$ |

## Generation 1000

Fitness $=492$

Time: $0 \quad$ Score: 0

| S |  | - |  |  |  |  | 1 | 1 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | - | - | 1 | 1 |  |  |  |  |
| $\square$ |  | 1 | $\bullet$ |  | 1 | 1 | 1 |  |  |
|  |  |  | 1 | $\bullet$ | - | 1 |  |  |  |
|  |  |  |  | 1 | 1 | 1 |  |  |  |
|  |  |  | $\square$ |  | 1 |  | 1 | $\square$ |  |
| 1 | 1 | $\square$ |  |  | 1 | 1 | 1 | 1 | - |
| 1 | 1 |  |  |  | 1 | 1 | 1 |  | - |
| $\square$ |  |  | 1 | , | , | - | - |  |  |
| 1 | - |  |  |  |  |  |  |  |  |

## Time: 1 Score: 0



Time: 2 Score: 10


Time: 3 Score: 10


Time: 4 Score: 20

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  | $\checkmark$ |  |  |  |  | $\square$ | $\bigcirc$ | $\bigcirc$ |
| 1. | - |  | 1 | $\square$ | 0 | 1 |  |  |  |  |
| 2 | - |  | $\square$ | $\square$ |  | 1 | 1 | $\square$ |  |  |
| 3 |  |  |  | $\square$ | - | $\square$ | 1 |  |  |  |
| 4 |  |  |  |  | 1 | 1 | 1 |  |  |  |
| 5 |  |  |  | $\square$ |  | $\square$ |  | 0 | 8 |  |
| 6 | $\bigcirc$ | 8 | 1 |  |  | 1 | 0 | 0 | - | $\square$ |
| 7 | $\square$ | - |  |  |  | $\square$ | - | - |  | $\square$ |
| 8 | $\triangle$ |  |  | $\triangle$ | $\square$ | $\square$ | $\bigcirc$ | $\square$ |  |  |
| 9 | $\square$ | $\triangle$ |  |  |  |  |  | $\square$ | $\triangle$ | $\triangle$ |

Time: 5 Score: 20

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  | $\checkmark$ |  |  |  |  | 1 | $\bigcirc$ | $\bigcirc$ |
| 1 |  |  | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |  |  |  |  |
| 3 | $\stackrel{9}{8}$ |  | 1 | 1 |  | $\bigcirc$ | $\triangle$ | 1 |  |  |
| 3 | - |  |  | - | 1 | 0 | 0 |  |  |  |
| 4 |  |  |  |  | - | 1 | - |  |  |  |
| 5 |  |  |  | - |  | $\bigcirc$ |  | $\square$ | 8 |  |
| 6 | 1 | $\bigcirc$ | $\bigcirc$ |  |  | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 0 |
| 7 | $\bigcirc$ | - |  |  |  | 1 | - | 1 |  | $\square$ |
| 8 | $\square$ |  |  | - | - | 1 | - | 1 |  |  |
| 9 | $\square$ | - |  |  |  |  |  | 1 | 0 | $\square$ |

Time: 6 Score: 20

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  | $\checkmark$ |  |  |  |  | 1 | $\bigcirc$ | 0 |
| 1 |  |  | $\bigcirc$ | - | - | 1 |  |  |  |  |
| 2 |  |  | - | - |  | 1 | - | - |  |  |
| 3 | 9 |  |  | $\square$ | $\square$ | $\bigcirc$ | $\square$ |  |  |  |
| 4 | $\cdots$ |  |  |  | $\square$ | $\bigcirc$ | $\square$ |  |  |  |
| 5 |  |  |  | 1 |  | $\square$ |  | $\square$ | $\bigcirc$ |  |
| 6 | 1 | $\bigcirc$ | - |  |  | 1 | - | $\square$ | $\bigcirc$ | $\square$ |
| 7 | - | $\square$ |  |  |  | 1 | - | $\square$ |  | $\square$ |
| 8 | $\square$ |  |  | 1 | 1 | 1 | 1 | $\square$ |  |  |
| 9 | $\square$ | $\square$ |  |  |  |  |  | 1 | $\bigcirc$ | $\square$ |

Time: 7 Score: 20

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  | 1 |  |  |  |  | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 1 |  |  | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |  |  |  |  |
| 2 |  |  | 1 | 1 |  | 1 | - | 1 |  |  |
| 3 |  |  |  | - | - | $\square$ | - |  |  |  |
| $4$ | 9 |  |  |  | - | $\bigcirc$ | - |  |  |  |
| 5 | $\pi$ |  |  | $\bigcirc$ |  | $\square$ |  | $\bigcirc$ | $\bigcirc$ |  |
| 6 | 0 | 1 | 1 |  |  | 1 | - | $\square$ | $\square$ | 1 |
| 7 | - | $\square$ |  |  |  | $\bigcirc$ | - | $\bigcirc$ |  | 1 |
| 8 | $\square$ |  |  | $\bigcirc$ | $\square$ | $\bigcirc$ | $\square$ | $\square$ |  |  |
| 9 | $\square$ | $\square$ |  |  |  |  |  | $\square$ | $\bullet$ | $\square$ |

Time: 8 Score: 20

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  | $\checkmark$ |  |  |  |  | 1 | $\bigcirc$ | $\bigcirc$ |
| 1 |  |  | 0 | - | $\square$ | 0 |  |  |  |  |
| 2 |  |  | $\bigcirc$ | $\bigcirc$ |  | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |  |  |
| 3 |  |  |  | 1 | 1 | $\bigcirc$ | 1 |  |  |  |
| 4 |  |  |  |  | 1 | 0 | - |  |  |  |
| $5$ | $9$ |  |  | 1 |  | $\square$ |  | $\square$ | $\bigcirc$ |  |
| 6 | - | 1 | $\bigcirc$ |  |  | $\bigcirc$ | $\square$ | $\square$ | $\bigcirc$ | $\square$ |
| 7 | 1 | $\square$ |  |  |  | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |  | $\square$ |
| 8 | - |  |  | - | $\square$ | 0 | - | $\square$ |  |  |
| 9 | $\square$ | $\bigcirc$ |  |  |  |  |  | $\square$ | $\square$ | $\square$ |

Time: $9 \quad$ Score: 30

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  | $\square$ |  |  |  |  | 1 | $\bigcirc$ | $\bigcirc$ |
| 1 |  |  | 1 | - | 1 | 0 |  |  |  |  |
| 2 |  |  | 0 | $\square$ |  | $\bigcirc$ | 1 | 1 |  |  |
| 3 |  |  |  | 1 | 1 | $\bigcirc$ | - |  |  |  |
| 4 |  |  |  |  | - | 0 | - |  |  |  |
| $5$ | $9$ |  |  | 1 |  | $\square$ |  | 1 | $\bigcirc$ |  |
| 6 |  | 1 | $\theta$ |  |  | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\square$ |
| 7 | 0 | $\square$ |  |  |  | 1 | - | 1 |  | $\square$ |
| 8 | - |  |  | - | - | 0 | - | 1 |  |  |
| 9 | $\square$ | $\bigcirc$ |  |  |  |  |  | $\square$ | $\square$ | $\square$ |

Time: 10 Score: 30

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  | $\square$ |  |  |  |  | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 1 |  |  | $\bigcirc$ | $\bigcirc$ | $\square$ | $\bigcirc$ |  |  |  |  |
| 2 |  |  | $\square$ | 1 |  | 0 | 1 | 1 |  |  |
| 3 |  |  |  | - | 1 | $\square$ | $\square$ |  |  |  |
| 4 |  |  |  |  | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |  |  |  |
| 5 |  |  |  | $\bigcirc$ |  | $\square$ |  | $\bigcirc$ | $\bigcirc$ |  |
| $6$ | $9$ | $\bigcirc$ | - |  |  | $\bigcirc$ | $\square$ | $\bigcirc$ | $\bigcirc$ | $\square$ |
| 7 | H | $\square$ |  |  |  | $\bigcirc$ | $\square$ | $\square$ |  | $\square$ |
| 8 | - |  |  | $\bigcirc$ | $\square$ | $\bigcirc$ | $\square$ | 1 |  |  |
| 9 | $\square$ | $\square$ |  |  |  |  |  | $\square$ | $\bigcirc$ | $\square$ |

Time: 11 Score: 40


Time: 12 Score: 40


Time: 13 Score: 50


Time: 14 Score: 50


Time: 15 Score: 60


Time: 16 Score: 60

| 0 |
| :--- |
| 0 |
| 0 | | 0 |
| :--- |

Time: 17 Score: 70
0

0 | 0 |
| :--- |
| 0 |

Time: 18 Score: 70


## Why Did The GA's Strategy Outperform Mine?

My Strategy

|  |  |  |  |  |  |  |  |  |  |  |
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|  |  |  |  |  |  |  |  |  |  |  |
| 1 |  |  | 9 |  |  |  |  |  |  |  |
| 2 |  | $\square$ | 1 |  | , | 1 |  |  |  |  |
| 3 |  |  |  |  |  | 1 |  |  |  |  |
| 4 |  |  |  |  |  |  |  | - |  |  |
| 5 |  |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |  | 1 |  |
| 1 |  |  |  |  |  |  |  | 1 |  |  |
| 8 |  |  |  |  |  |  |  |  |  |  |
| 9 | $\square$ |  |  |  |  |  |  |  |  |  |


|  |  | 2 |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |
| 1 |  |  | 9 |  |  |  |  |  |  |  |
| 2 |  | $\square$ | - |  | , | 1 |  |  |  |  |
| 3 |  |  |  |  |  | 1 |  |  |  |  |
| 4 |  |  |  |  |  |  |  | - |  |  |
| 5 |  |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |  | 1 |  |
| 1 |  |  |  |  |  |  |  | 1 |  |  |
| 8 |  |  |  |  |  |  |  |  |  |  |
| 9 | $\square$ |  |  |  |  |  |  |  |  |  |


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|  |  |  |  |  |  |  |  |  |  |  |
| 1 |  | 9 |  |  |  |  |  |  |  |  |
| 2 |  | 1 |  | 1 |  | 1 |  |  |  |  |
| 3 |  |  |  |  |  | $\pm$ |  |  |  |  |
| 4 |  |  |  |  |  |  |  | - |  |  |
| 5 |  |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |  | 1 |  |
| 1 |  |  |  |  |  |  |  | 1 |  |  |
| 8 |  |  |  |  |  |  |  |  |  |  |
| 9 | $\square$ |  |  |  |  |  |  |  |  |  |


|  |  |  | 3 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |
| 1 |  | 9 |  |  |  |  |  |  |  |
| 2 |  | $\checkmark$ |  | $\square$ | $\bullet$ |  |  |  |  |
| 3 |  |  |  |  | $\checkmark$ |  |  |  |  |
| 4 |  |  |  |  |  |  | $\square$ |  |  |
| 5 |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  | 1 |  |
| 7 |  |  |  |  |  |  | 1 |  |  |
| 8 |  |  |  |  |  |  |  |  |  |
| 9 | $\square$ |  |  |  |  |  |  |  |  |


|  |  |  | 3 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |
| 1 |  | 9 |  |  |  |  |  |  |  |
| 2 |  | $\checkmark$ |  | $\square$ | $\bullet$ |  |  |  |  |
| 3 |  |  |  |  | $\checkmark$ |  |  |  |  |
| 4 |  |  |  |  |  |  | $\square$ |  |  |
| 5 |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  | 1 |  |
| 7 |  |  |  |  |  |  | 1 |  |  |
| 8 |  |  |  |  |  |  |  |  |  |
| 9 | $\square$ |  |  |  |  |  |  |  |  |

The GA's Evolved Strategy

|  |  |  |  |  |  |  |  |  |  |  |
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| 1 |  |  | 9 |  |  |  |  |  |  |  |
| 2 |  | $\square$ | 1 |  | , | 1 |  |  |  |  |
| 3 |  |  |  |  |  | 1 |  |  |  |  |
| 4 |  |  |  |  |  |  |  | - |  |  |
| 5 |  |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |  | 1 |  |
| 1 |  |  |  |  |  |  |  | 1 |  |  |
| 8 |  |  |  |  |  |  |  |  |  |  |
| 9 | $\square$ |  |  |  |  |  |  |  |  |  |



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|  |  |  |  |  |  |  |  |  |  |  |
| 1 |  | 9 |  |  |  |  |  |  |  |  |
| 2 |  | 1 | - | - | E |  |  |  |  |  |
| 3 |  |  |  |  | 1 |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  | - |  |  |
| 5 |  |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |  | 1 |  |
| 7 |  |  |  |  |  |  |  | 1 |  |  |
| 8 |  |  |  |  |  |  |  |  |  |  |
| 9 | 1 |  |  |  |  |  |  |  |  |  |


|  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | 9 |  |  |  |  |  |  |
|  |  |  |  | 1 | - | 1 |  |  |  |  |
| 3 |  |  |  |  |  | $\square$ |  |  |  |  |
| 4 |  |  |  |  |  |  |  | 1 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | 1 |  |
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| 3 |  |  |  |  | $\square$ |  |  |  |  |
| 4 |  |  |  |  |  |  | 1 |  |  |
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| 6 |  |  |  |  |  |  |  | $\square$ |  |
| 1 |  |  |  |  |  |  | 1 |  |  |
| 8 |  |  |  |  |  |  |  |  |  |
| 9 | $\bullet$ |  |  |  |  |  |  |  |  |


|  | 1 | 2 |  |  |  |  |  |  |  |
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|  |  |  |  |  |  |  |  |  |  |
| 1 |  |  |  | 9 |  |  |  |  |  |
| 2 |  |  |  | 5 | $\square$ |  |  |  |  |
| 3 |  |  |  |  | $\square$ |  |  |  |  |
| 4 |  |  |  |  |  |  | - |  |  |
| 5 |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  | 1 |  |
| 1 |  |  |  |  |  |  | 1 |  |  |
| 8 |  |  |  |  |  |  |  |  |  |
| 9 | $\square$ |  |  |  |  |  |  |  |  |


|  | 1 | 2 | 3 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |
| 1 |  |  |  | 9 |  |  |  |  |  |
| 2 |  |  |  | 1 | $\square$ |  |  |  |  |
| 3 |  |  |  |  | $\square$ |  |  |  |  |
| 4 |  |  |  |  |  |  | $\square$ |  |  |
| 5 |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  | 1 |  |
| 7 |  |  |  |  |  |  | 1 |  |  |
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| 4 |  |  |  |  |  |  | $\square$ |  |  |
| 5 |  |  |  |  |  |  |  |  |  |
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| 7 |  |  |  |  |  |  | 1 |  |  |
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|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
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| 0 |  |  |  |  |  |  |  |  |  |  |
| 1 |  |  |  |  |  | $\mathscr{8}$ |  |  |  |  |
| 2 |  |  |  |  |  | $\triangle$ |  |  |  |  |
| 3 |  |  |  |  |  | $\bigcirc$ |  |  |  |  |
| 4 |  |  |  |  |  |  |  | $\bigcirc$ |  |  |
| 5 |  |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |  | 1 |  |
| 7 | $\square$ |  |  |  |  |  |  | $\square$ |  |  |
| 8 |  |  |  |  |  |  |  |  |  |  |
| 9 |  | $\square$ |  |  |  |  |  |  |  |  |


|  |  |  | 2 | 3 |  |  | 6 |  |  |  |
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| 4 |  |  |  |  |  |  |  | $\square$ |  |  |
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